Electrical conductivity measurements in liquid metals by rotational technique

S. I. BAKHTIYAROV, R. A. OVERFELT Space Power Institute, 231 Leach Center, Auburn University Auburn, AL 36849-5320

The rotational contactless inductive measurement technique has been developed to measure the electrical conductivity of liquid metals. This method is based on the phenomena when a conductor material rotates in a magnetic field, circulating eddy currents are induced and generate a damping torque proportional to the electrical resistivity of the material. The technique was tested to measure the conductivity of five conductors and one low melting composite (LMA-158). © 1999 Kluwer Academic Publishers

1. Introduction

The mechanical properties of metals depend strongly on physical properties (such as, viscosity, electrical conductivity, density, surface tension, etc.) of liquid metals. The optimization and control of many metallurgical processes requires the knowledge of these and many other physical properties. Electrical conductivity is a basic physical factor in evaluation and designing new alloys, and provides valuable information about the structure of molten metals.

The electrical conductivity is an ability of the material to carry electrical current in the presence of an applied voltage. It is expressed in terms of the current density (A m⁻²) per unit electric field (V m⁻¹). Electrical conductivity (σ_e) is the reciprocal of electrical resistivity (ρ_e):

$$\sigma_e = 1/\rho_e. \tag{1}$$

Electrical resistivity is a measure of the resistance of a unit quantity of a given material. Electrical resistivity may be expressed in terms of mass (δ_e) or volume (ρ_e) resistivities computed from the following formulas, respectively:

$$\delta_e = R\rho\pi D^2/4L,\tag{2}$$

$$\rho_e = R\pi D^2 / 4L,\tag{3}$$

where R is a resistance, ρ is a material density, D and L are diameter and length of the sample, respectively. The International Annealed Copper Standard (IACS) is the internationally accepted value for the resistivity of annealed copper of 100% conductivity.

The methods of electrical conductivity measurements can be categorized into two groups:

- direct resistance measurements and
- contactless inductive measurements.

In direct resistance measurement techniques high melting metals (platinum, molybdenum, etc.) are used as electrodes. In aggressive metal melts they are subject to solution attack. Therefore, this methods are applicable for poor conductors and for calibration of the contactless inductive technique. High melting noble metals are used as electrodes in the direct measurement techniques.

As a direct technique, four-probe potentiometric method has been developed to measure electrical conductivity of molten metals [1, 2]. In this method, at a constant current density the potential drop across the molten sample in a capillary tube of a known cross-section and length is measured. The probe cell has to be calibrated using a liquid (usually mercury) of known electrical conductivity. A selection of the proper material for the capillary cell and the electrodes is a main problem in this method. Dissolution of electrodes and chemical reaction between them may happen. Hence, an improved four-probe method has been developed later [3]. This method uses solid electrodes made of identical material to the molten sample.

The contactless inductive measurement technique excludes the dissolution, chemical reaction and contact resistance phenomena. The method is based on the phenomena that when a metal sample moves in a magnetic field (or magnetic field rotates around the sample), circulating eddy currents are induced in the sample. These currents generate a damping torque proportional to the electrical conductivity of the sample. The rotating magnetic field method to measure the electrical conductivity of molten metals and alloys have been used by Samarin [4] and Ono and Yagi [5]. The advantage of the method is the possibility of measuring at the same time the viscosity of the molten sample by the damping of the torsional oscillation of the cell. Overfelt et al. [6] successfully utilized the oscillating vessel technique for measuring molten metal viscosity over the temperature range of 1350 to 1475 $^{\circ}$ C.

The objective of this study is to develop the rotational technique to measure an electrical conductivity of molten metals and to test this method on metals and alloys of known electrical conductivity.

TABLE I Sizes and properties of samples tested

Sample	D (cm)	ΔL (cm)	ρ (g/cm ³)	$R_e \ (\mu\Omega)$	$ ho_e \ (\mu\Omega{ m cm})$	$\sigma_e \ (\mu\Omega{ m cm})^{-1}$	σ_e (IACS %)
Al-1	0.965	2.540	2.692	11.69	3.366	0.297	51.22
Al-2	0.965	2.535	2.805	13.85	3.996	0.250	43.15
Brass	0.953	2.746	8.387	28.59	7.416	0.135	23.25
Cu	0.950	2.350	8.892	5.717	1.724	0.580	100.0
LMA158	0.965	2.385	9.460	57.31	17.57	0.057	9.62
S/Steel	0.960	2.591	8.321	383.1	107.0	0.009	1.611

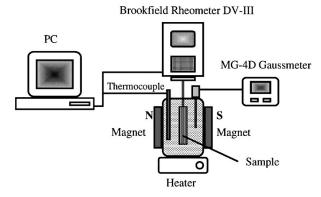


Figure 1 Experimental apparatus used for electrical conductivity measurements of solid and molten metals.

2. Experimental apparatus and procedure

A schematic of the experimental apparatus to measure an electrical conductivity of solid and molten metals and alloys is shown in Fig. 1. Computer controlled Brookfield rheometer Model DV-III has been used to provide a constant rotational speed to metal sample and to measure the torque precisely. This rheometer allows to make torque measurements (0 to 673.7 dyn cm) at constant speeds from 0 to 250 rpm in 0.1 rpm increments. Rheometer has RS232 Compatible serial Port for use with attached PC. The metal sample was placed inside the acrylic tube (crucible) attached to the spindle of the rheometer. The acrylic tube (9.525 mm diameter and 45 mm length) with sample has been placed in water bath. The magnetic field has been generated by two neodymium permanent magnets. MG-4D gaussmeter (Walker Scientific Inc.) operated on the Hall-effect principle has been used to measure the magnetic field strength. It provides DC and AC field readings from $\pm 10^{-5}$ to ± 2 T with 0.1% resolution. Changing the distance between the magnets we could obtain a magnetic field of desirable strength. Fig. 2 shows the variation of the magnetic field strength with the distance between the magnets.

As a test sample we used different metals and composites. The sizes and some properties of these samples at $t=24\,^{\circ}\mathrm{C}$ are shown in Table I. As a composite material we used low melting alloy LMA-158. Percentage composition of the alloy and electrical conductivity of the components are shown in Table II. The melting temperature of this composite is 70 °C, hardness is 12 by Brinell, thermal expansion is 0.27%. This composite sample can be melted under hot water and can be recovered and tested over again.

The electrical resistance of the samples has been measured by using 4300B Digital Micro-ohmmeter

TABLE II LMA-158 components and their electrical properties

LMA-158, components	Weight, (%)	$ ho_e \ (\mu\Omega\ { m cm})$	$\sigma_e \ (\mu\Omega\ { m cm})^{-1}$
Bismuth	50.0	107.00	0.00935
Lead	26.7	19.30	0.05181
Tin	13.3	10.10	0.09901
Cadmium	10.0	6.73	0.14859

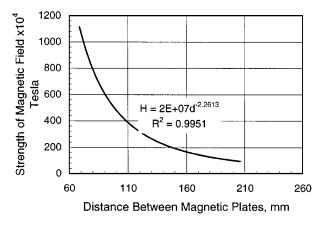


Figure 2 Variation of magnetic field strength with distance between magnetic plates.

(Valhalla Scientific, Inc.). The Kelvin four-terminal configuration of this ohmmeter eliminates errors caused by test lead and contact resistance, which in many applications can exceed the value of the load, by several orders of magnitude.

To avoid the end effects at the ends of the test samples, the difference in resistance for two samples of the same material and diameter but of different lengths $(\Delta L = L_2 - L_1)$ was measured at the same temperature. This electrical resistance difference is the resistance across a test conductor whose length is the difference between the lengths of the two samples used (e.g. ΔL), which is free of all end effects.

To estimate the end effects we measured the elecrical resistance of three copper cylinders of 1.114 cm diameter and different lengths using 4300B Digital Microohmmeter (Table III). Results of measurements are shown in Fig. 3. As seen from this figure, there is discrepancy in electrical conductivity data for samples of different lengths. The electrical conductivity decreases with increasing the sample length. Hence, one would assume a significant contribution of ends effects on measured values of electrical resistance. No variations in electrical conductivity data were observed when we

TABLE III Estimation of end effects by measuring the electrical conductivity of copper cylinders

D(cm)	L(cm)	$R(\mu\Omega)$	$\rho(\mu\Omega~{\rm cm})$	$\sigma 1/\mu\Omega$ cm
1.114 1.114 1.114 1.114 1.114	$L_1 = 13.65$ $L_2 = 9.1$ $L_3 = 5.8$ $L_1 - L_2 = 4.55$ $L_1 - L_3 = 7.85$		1.648 1.584 1.478 1.776 1.774	0.607 0.631 0.677 0.563 0.564
1.114	$L_2 - L_3 = 3.30$	$R_2 - R_3 = 6.0$	1.771	0.565

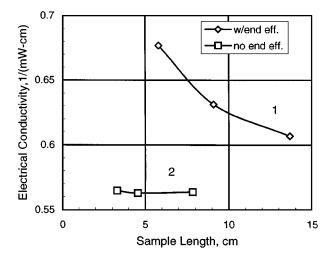


Figure 3 End effects on the electrical conductivity measurements by micro-ohmmeter (copper, D = 1.114 cm, t = 24 °C).

avoided the end effects by calculating the conductivity by the resistance across a sample whose length is the difference between the lengths of the two samples.

3. Results and discussion

From both scientific and practical view of points it is important to know the order of the magnetic Reynolds numbers defined as

$$Re_m = \omega \sigma \mu_0 R^2, \tag{4}$$

where μ_0 is a magnetic permeability. Fig. 4 shows the variation of the magnetic Reynolds number with the angular velocity for different test samples. As seen from the figure, $\text{Re}_m \ll 1$, which means that laminar flow regime has been treated in our experiments.

Fig. 5 shows the variation of the damping torque (M)caused by induced circulation eddy currents and measured at constant rotational speed ($\omega = 250$ rpm) by using Brookfield rheometer, with electrical conductivity of the tested material exposed to magnetic field of different strength (from 0.00879 to 0.103 T). The values of the electrical conductivity of the materials have been determined by direct resistance measurement technique using MG-4D gaussmeter described above. As seen from this figure, the magnetic field strength plays a predominant role in values of the damping torque. Increasing the strength of the field for a given sample results in increasing the damping torque significantly. The predictions of the numerical model developed by Spitzer et al. [7] are also shown in Fig. 5. There are qualitative and quantitative agreements with the data

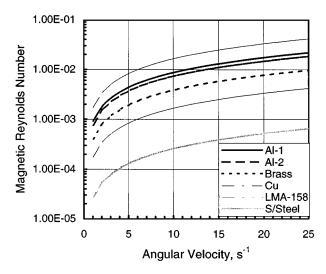


Figure 4 Variation of the magnetic Reynolds number with the angular velocity for different test samples.

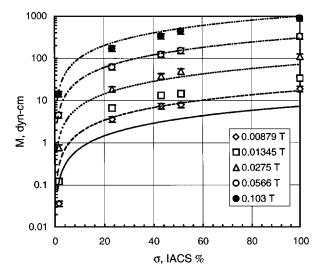


Figure 5 Variation of the damping torque caused by the induced circulating eddy currents with the electrical conductivity of the material (ω = 250 rpm); simulations by Spitzer *et al.* [7]: — 0.00879 T;— — 0.01345 T; - - - 0.0275 T;— - — 0.0566 T; — - - — 0.103 T.

except for relatively small values of the magnetic field strength.

The relationship between a damping torque and an electrical conductivity of the material can be expressed by polynomial of power two

$$M = a\sigma^2 + b\sigma + c, (5)$$

where a, b and c are coefficients of the polynomial and are functions of the magnetic field strength. The coefficients of the polynomial as functions of the magnetic field strength are shown in Fig. 6. Applying regression analysis, a power equation was found satisfactorily to describe these relationships ($R^2 > 0.994$).

For calibration tests it is convenient to have expression for the electrical conductivity as a function of the damping torque measured on Brookfield rheometer. A polynomial expression also can be applied for this relationship. For the magnetic field of known strength the coefficients of these polynomials can be determined from Fig. 7.

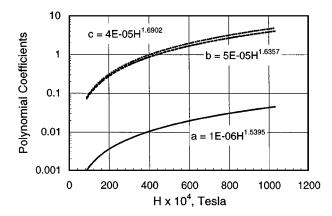


Figure 6 Variation of the polynomial coefficients as functions of the magnetic field strength.

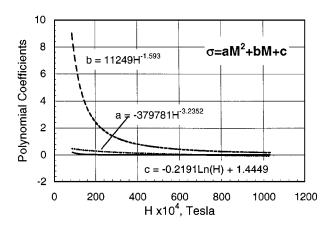


Figure 7 Variation of the polynomial coefficients as functions of the magnetic field strength.

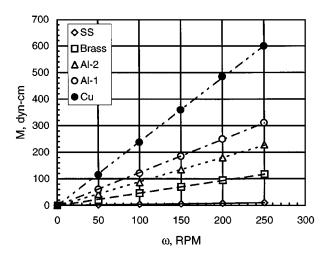


Figure 8 Variation of the damping torque caused by the induced circulating eddy currents with the angular velocity of rotation (H=0.075 T; (\Leftrightarrow) 100 IACS%; (\square) 51.22 IACS%; (\triangle) 43.15 IACS%; (\bigcirc) 23.25 IACS%; (\bullet) 1.611 IACS%).

Variation of damping torque caused by the induced circulating eddy currents with the angular velocity of sample rotation at the magnetic field strength $H=0.075\,\mathrm{T}$ for tested samples is represented in Fig. 8. The results show that there is linear relationship between damping torque and angular velocity of rotation. The linear relationship is consistent with the predictions made by Regel [4].

Prior to study the electrical conductivity of high melting conductor materials, it is methodologically interest-

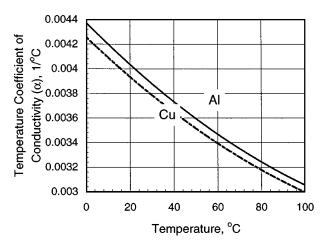


Figure 9 Variation of the temperature coefficient of conductivity with temperature for aluminum and copper.

ing to measure the electrical conductivity changes in tested materials over moderate ranges of temperature (such as $100\,^{\circ}$ C) by proposed method. Also many engineering simulations require considering the change in conductivity with change in temperature.

It is well known that over moderate ranges of temperature, the change of electrical conductivity is inversely proportional to the change of temperature

$$\sigma_e = \sigma_{e1}/[1 + \alpha(t - t_1)], \tag{6}$$

where σ_e and σ_{e1} are electrical conductivities at temperatures t and t_1 , respectively; α is a temperature coefficient of electrical conductivity. The temperature coefficient is a function of the temperature [8]

$$\alpha = 1/[1/\alpha_1 + (t - t_1)],\tag{7}$$

where α and α_1 are temperature coefficients at temperatures t and t_1 , respectively. Fig. 9 shows the variation of the temperature coefficients of conductivity over moderate ranges of temperature for aluminum and copper.

The experimental apparatus described above allowed us to accomplish electrical conductivity measurements of selected materials over moderate ranges of temperature (up to 90 °C). Fig. 10 shows the variation

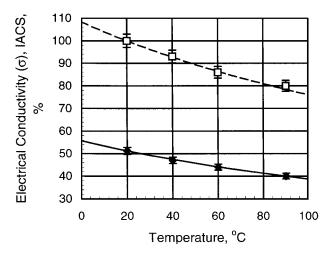


Figure 10 Variation of the electrical conductivity of aluminum-1 and copper with temperature at 0.0522 T magnetic field strength (\square Cu; \spadesuit Al-1).

of the measured values of the electrical conductivity of two samples (Aluminum-1 and Copper) with the temperature at fixed strength of magnetic field $(0.0522 \,\mathrm{T})$. The predictions of the electrical conductivity for these materials using Equation 6 are also shown in Fig. 10. There seems to be good agreement for the all temperatures range. However, we remark that over wider ranges of temperature the inversely linear relationship of the formula (6) is not usually applicable, and the formula then becomes a series involving higher powers of t [8]. Moreover, Equations 6 and 7 take no account of the change in dimensions with change in temperature and therefore applies to the materials of constant mass. According to regression analysis the experimental data best can be described by the polynomial of power two

$$\sigma_e = dt^2 + et + f, (8)$$

where d, e and f are coefficients of the polynomial. Their values for tested materials are given in Table IV.

The change in electrial conductivity of LMA-158 composite with change in temperature over the range including the melting point of the material $(70 \,^{\circ}\text{C})$ is shown in Fig. 11. This relationship also is best described

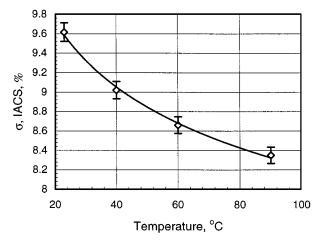


Figure 11 Variation of the electrical conductivity of LMA-158 with temperature at $\omega=250$ rpm and H=0.0522 T magnetic field strength.

TABLE IV The values of the coefficients of polynomial (8)

Material	d	e	f
Aluminum-1	0.0008	-0.2462	55.761
Copper	0.0018	-0.4917	109.26
LMA-158	0.0003	-0.0476	10.557

by formula (8). The polynomial coefficients of this relationship are given in Table IV.

4. Conclusions

The rotational technique to measure an electrical conductivity of molten metals has been developed. The method was used to measure the electrical conductivity of several metals and low melting alloy LMA-158 over moderate ranges of temperature (up to 90 $^{\circ}$ C) including the melting point of the composite.

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